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(Bejan's) early vs. late regimes method applied to entropy generation in one-dimensional conduction

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Abstract

In this paper, we treat the unsteady entropy generation rate due to an instantaneous internal heat generation in a solid slab. Following the basic ideas developed by Bejan [Heat Transfer, Wiley, 1993], we conduct a multiple-scale analysis identifying the "early" and "late" regimes to derive, in a very simple way, the nondimensional unsteady temperature profile for small values of the Biot number, *Bi*. In consequence, the nondimensional spatial average entropy generation rate per unit volume, Φ and the corresponding average steady-state entropy generation rate, Ψ, were evaluated for different values of the nondimensional heat generation parameter β. This parameter represents physically the ratio of the temperature of the solid slab (due to the internal heat generation) to the fluid temperature. We show that for the assumed values of this parameter β, the nondimensional temperature and entropy generation rate variables present a very sensible dependence between both parameters, indicating a direct relationship between the basic heat transfer mechanics: heat conduction, heat convection and internal heat generation.

Keywords: Entropy generation; Unsteady heat conduction; Solid slab; Multiple-scale analysis; Internal heat generation; Lumped model; Transient; Conduction

1. Introduction

Nowadays, the theoretical method of entropy generation minimization (EGM) has been widely used in the specialized literature to treat real thermodynamics systems with the simultaneous presence of external and internal irreversibilities. In general, these irreversibilities are related with different effects like heat and mass transfer, fluid flow, turbulence, chemical reactions, etc. and novel techniques are required to control optimally the above thermodynamic imperfections. In a recent book, Bejan [1] developed these ideas to emphasize that the basic principles of thermodynamics, heat transfer and fluid mechanics can combine in a fundamental method to reach a unified treatment more easier and effective to use in engineering research. In the above reference is pointed out the practical importance of time-depend heating and cooling operations in conventional applications as refrigeration and power generation. These examples illustrate clearly the direct connection between entropy generation and the heat transfer and fluid mechanics disciplines. However, the influence of transient effects taking into account entropy generation for simple models has not been treated enough, in spite of exist abundant information related with the physical importance of these effects (see for instance the books of Kudryavtsev [2], Ozisik [3] and Patankar [4]). On the other hand, many industrial equipment are subjected to transient changes in the thermal loading conditions, affecting suddenly the thermal efficiency and performance. The above is appreciated during the transient period of start-up, shutdown or any off-normal operation in such systems. Very illustrative examples can be found in [5].

Recognizing the fundamental and practical relevance of the transient entropy generation effects, we have selected a

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Nomenclature

Bi	Biot number, $= hL/k$
c	specific heat of the solid slab
h	convective heat transfer coefficient of the fluid
L	half-thickness of the solid slab
q	internal heat generation
s	nondimensional slow time scale, defined in Eq. (18)
t	physical time
T	temperature
T_i	characteristic temperature of the fluid
x	Cartesian coordinate

Greek symbols

α	thermal diffusivity of the solid slab
β	nondimensional heat generation parameter, defined in Eq. (13)
λ	thermal conductivity of the solid slab
ρ	density of the solid slab
σ	nondimensional fast time scale, defined in Eq. (18)
θ	nondimensional temperature of the solid slab, defined in Eq. (7)
χ	nondimensional coordinate, defined in Eq. (7)
τ	nondimensional time scale, defined in Eq. (7)

simple model to analyze this kind of situations: the unsteady heat conduction in a solid slab due to the presence of an internal heat generation. This simple case permit us to illustrate basic aspects related with internal irreversibilities due to a direct heat generation in nuclear reactors [6]. For simplicity, we developed an analysis for small values of the Biot number, Bi, which constitutes a dimensionless form of the convective heat transfer coefficient, h. As it is well know, the Biot number measures the competition between the heat conduction in the solid body to the convective heat with the neighboring fluid. For $Bi \ll 1$ and bodies of simple geometries, the lumped model is really a good approximation; however, Cortéz et al. [7] studying the classical problem of unsteady heat conduction in bodies of elementary geometry (large slab, long cylinder and sphere) have proved that the lumped calculation follows as a particular case from the infinite series solution of the general distributed model. In a very recent article, Ibáñez et al. [8] analyzed the minimization of the entropy generation of a solid slab with a steady-state internal heating. The solid slab is placed horizontally in such a way that the external surfaces of the solid are exposed to convective ambients with different Biot numbers. They obtained analytically for a wide range of Biot numbers, the global entropy generation rate and the corresponding minimum values of this function. For instance, these authors controlling the Biot number in one of the surfaces, an optimum Biot number for the second surface that minimizes the global entropy generation rate was found.

In order to elucidate the role of transient entropy generation effects, we treat in the present work, the unsteady heat conduction process in a solid slab due to the presence of an internal heat generation. For simplicity, we consider that solid slab is exposed to an external ambient characterized by a uniform convective heat transfer coefficient, h. The energy equation for the slab is derived by taking into account small values of the Biot number. We use multiple-scale techniques to show that the temperature profile and entropy generation are controlled by two nondimensional parameters: Bi, and β. The Biot number Bi measures the competition between the heat conduction through the solid slab and the convective heat to the fluid. On the other hand, β is the ratio of the characteristic temperature of the slab to the characteristic fluid temperature. We anticipate that the unsteady heat conduction entropy generation effects of the slab are fully controlled by these parameters, which dictate the different unsteady heat transfer regimes. Therefore, the corresponding entropy generation rate is predicted with the influence of both nondimensional parameters.

2. Order of magnitude analysis and formulation

The physical model under study is shown in Fig. 1. We consider a solid slab of half-thickness L, thermal conductiv-

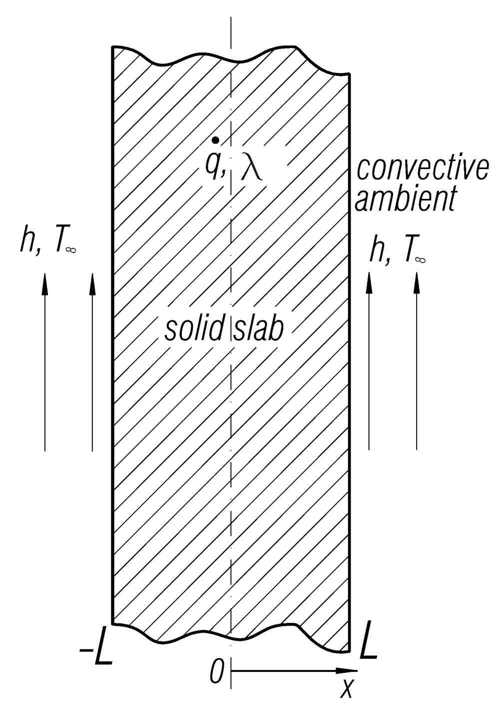

Fig. 1. Sketch of the physical model.

ity λ, density ρ, specific heat c; initially in thermodynamic equilibrium with a surrounding fluid at temperature T_i. Suddenly, the solid slab experiences a uniform internal heat generation rate per unit volume $\dot{q}$, yielding on the external surfaces of the slab a convective heat transfer process, which is controlled basically by the convective heat transfer coefficient h. We identify two natural time scales in this problem: characteristic thermal penetration or thermal diffusion time at the solid slab, $t_D \sim L^2/\alpha$ and the characteristic convective heat response time, $t_C \sim \rho c L/h$. Here, α is the thermal diffusivity of the solid slab defined as $\alpha = \lambda/\rho c$. The ratio of both characteristic times defines the well-known Biot number:

$$Bi = \frac{t_D}{t_C} = \frac{hL}{\lambda} \tag{1}$$

therefore for small values of Biot numbers, the unsteady heat conduction process in the solid slab with internal heat generation is always controlled by the characteristic time scale t_C. This means that t_C represents the appropriate time scale to nondimensionalize the energy equation of the solid slab. On the other hand, the suitable spatial scale for this heat transfer process corresponds directly to the half-thickness of the slab; thus $x \sim L$, where x is the Cartesian coordinate needed to define the coordinate system. Furthermore, the characteristic temperature drop ΔT_c of the system is easily obtained through an energy balance between the heat diffusion term and the internal heat generation term, i.e.:

$$\frac{\Delta T_c}{L^2} \sim \frac{\dot{q}}{\lambda} \tag{2}$$

With this set of characteristic geometric and physic scales, the energy equation of the solid slab can be now adequately rewritten in a dimensionless form. However, first we show the corresponding energy equation in dimensional variables, and assuming uniform thermophysical properties, the form of the transient heat equation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\lambda} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{3}$$

with the initial condition

$$t = 0: \quad T = T_i \tag{4}$$

and the boundary conditions:

$$x = 0: \quad \frac{\partial T}{\partial x} = 0 \tag{5}$$

$$x = L: \quad -\lambda \frac{\partial T}{\partial x} = h(T - T_i) \tag{6}$$

It is should be noted that Eq. (5) represents the symmetry condition at the origin ($x = 0$) of the coordinate system, meanwhile Eq. (6) is the classical convective condition at the external surface of the solid slab ($x = L$). The symmetric condition (5) permit us to analyze only the temperature distributions for positive values of the coordinate x.

Introducing the dimensionless variables

$$\chi = \frac{x}{L}, \qquad \tau = \frac{t}{(\rho c L/h)}, \qquad \theta = \frac{T - T_i}{\Delta T_c} \tag{7}$$

therefore the nondimensional energy equation, Eq. (3), can be written as

$$\frac{\partial^2 \theta}{\partial \chi^2} + Bi = Bi \frac{\partial \theta}{\partial \tau} \tag{8}$$

with the initial and boundary conditions

$$\tau = 0: \quad \theta = 0 \tag{9}$$

$$\chi = 0: \quad \frac{\partial \theta}{\partial \chi} = 0 \tag{10}$$

and

$$\chi = 1: \quad \frac{\partial \theta}{\partial \chi} = -Bi\,\theta \tag{11}$$

The linear system of Eqs. (8)–(11) admits solutions for all possible values of the Biot numbers. In fact, classical solutions have been obtained in the past by conventional techniques and can be found elsewhere [9]. However, we want to illustrate the simplicity of the solution using multiple-scale analysis, because it is possible to derive a simple and closed formula for the slab temperature and easy to use in different situations. In particular, one reason to use multiple-scale analysis comes from the singular behavior of Eqs. (8)–(11): for small values of the Biot number, the right-hand side of Eq. (8) is negligible unless that $\tau \sim Bi$. In this "early" regime (see Bejan [10]), the transient term of the temperature must be retained. Otherwise, for $\tau \sim O(1)$, which defines the "late" regime, the steady-state regime is hold. Therefore, the introduction of these scales permit us to develop a simple solution of the temperature profile. On the other hand, we know that in the limit of $Bi \ll 1$, we must recover the classical lumped solution. In the opposite case, for large values of Biot number is indispensable to implement other regular perturbation analysis, which is not presented here.

In order to complete the formulation of the problem, the theorem of local entropy generation rate per unit volume, in dimensionless form can be written as [1]:

$$\Phi_s = \frac{\beta}{(1 + \beta\theta)^2} \left(\frac{\partial \theta}{\partial \chi} \right)^2 \tag{12}$$

where the nondimensional local volumetric rate of entropy generation Φ_s and the internal heat generation parameter β are defined as:

$$\Phi_s = \frac{\dot{S}_g'''}{(k/L^2)} \quad \text{and} \quad \beta = \frac{\dot{q} L^2}{\lambda T_i} \tag{13}$$

where $\dot{S}_g'''$ ($\mathrm{W \cdot m^{-3} \cdot K^{-1}}$) is the local volumetric rate of entropy generation in physical units and k/L^2 represents the characteristic entropy generation rate per unit volume. The system of Eqs. (8)–(12) should provide

$$\theta = \theta(\chi, \tau, Bi) \quad \text{and} \quad \Phi_s = \Phi_s(\chi, \tau, Bi, \beta) \tag{14}$$

Finally, is a common place in EGM analysis to use average formulas by entropy generation rates. Thus, we define the following nondimensional spatial average entropy generation rate per unit volume as:

$$\Phi = \Phi(\tau, Bi, \beta) = \int_0^1 \Phi_s(\chi, \tau, Bi, \beta)\,\mathrm{d}\chi \tag{15}$$

the advantage of this relationship is that in many engineering applications, we need to obtain the dependence of a global entropy generation rate only as a function of the involved nondimensional control parameters, avoiding the dependence on the specific geometry. This aspect has widely been reported by Bejan [1]. In addition, we also anticipate for $\tau \gg 1$ the function Φ_s (for fixed values of Bi and β) reaches a steady-state solution, which is represented as $\Phi_{ss} = \Phi_{ss}(\chi, Bi, \beta)$ and in this case, we define the corresponding average steady-state entropy generation rate

$$\Psi = \int_0^1 \Phi_{ss}(\chi, Bi, \beta)\,\mathrm{d}\chi \tag{16}$$

Both functions Φ and Ψ are evaluated numerically and the corresponding results are present in the last section.

3. Asymptotic solution for $Bi \ll 1$

For very small values of the Biot number Bi, the nondimensional temperature of the plate, θ, changes very little (of order Bi) in the spatial coordinate χ. However, in this limit the solution is singular; thus, two time scales appear in this problem: A slow time scale of order unity in τ controlling the global transient response of the plate and a rapid time scale, σ, of order Bi at the beginning process, as was previously commented. This rapid transient is due to that the initial condition for the temperature θ is lost for $Bi \ll 1$. This heat transfer problem can be studied using multiple scale analysis in the asymptotic limit $Bi \ll 1$, assuming the following expansion for the nondimensional wall temperature

$$\theta(\tau, \chi) = \sum_{j=0}^{\infty} B_i^j \theta_j(s, \sigma, \chi) \tag{17}$$

where s and σ are the slow and fast scales defined as,

$$s = \tau\left(1 + B_i\omega_1 + B_i^2\omega_2 + \cdots\right), \qquad \sigma = \tau / B_i \tag{18}$$

Introducing (17) and (18) into Eq. (8), we obtain after collecting terms of the same power of B_i, the following sets of equations:

$$\frac{\partial^2\theta_0}{\partial\chi^2} = \frac{\partial\theta_0}{\partial\sigma} \tag{19}$$

$$\frac{\partial^2\theta_1}{\partial\chi^2} + 1 = \frac{\partial\theta_1}{\partial\sigma} + \frac{\partial\theta_0}{\partial s} \tag{20}$$

$$\frac{\partial^2\theta_2}{\partial\chi^2} = \frac{\partial\theta_2}{\partial\sigma} + \frac{\partial\theta_1}{\partial s} + \omega_1\frac{\partial\theta_0}{\partial s} \tag{21}$$

with the following initial and boundary conditions:

$$\theta_0(\chi, 0, 0) = 0, \qquad \theta_i(\chi, 0, 0) = 0 \quad \text{for } i = 1, 2, \ldots \tag{22}$$

$$\left.\frac{\partial\theta_i}{\partial\chi}\right|_{\chi=0} = 0 \quad \text{for } i = 0, 1, 2, \ldots \tag{23}$$

and

$$\left.\frac{\partial\theta_0}{\partial\chi}\right|_{\chi=1} = 0, \qquad \left.\frac{\partial\theta_i}{\partial\chi}\right|_{\chi=1} = -\theta_{i-1} \quad \text{for } i = 1, 2, 3, \ldots \tag{24}$$

Eq. (19) can be integrated from $\chi = 0$ to $\chi = 1$ and applying the corresponding boundary conditions, we obtain that θ_0 is not a function of the short time scale and the longitudinal coordinate χ, depending only on the large time scale s, $\theta_0 = \theta_0(s)$. This function can be found after integrating the following higher-order equation (20) with the corresponding boundary conditions:

$$\frac{\partial}{\partial\sigma}\int_0^1 \theta_1\,\mathrm{d}\chi = \int_0^1 \frac{\partial^2\theta_1}{\partial\chi^2}\mathrm{d}\chi - \int_0^1 \mathrm{d}\chi - \frac{\partial}{\partial s}\int_0^1 \theta_0\,\mathrm{d}\chi \tag{25}$$

The left-hand side of Eq. (25) has to be zero in order to avoid secular terms for θ_1 and simplifying the other terms of the right-hand side, results a first-order differential equation for θ_0, given as

$$0 = -\theta_0 + 1 - \frac{\partial\theta_0}{\partial s} \tag{26}$$

In this form, using the initial condition, $\theta_0(0) = 1$, the solution for θ_0 is then given by

$$\theta_0(s) = 1 - \exp(-s) \tag{27}$$

which corresponds to the well-known transient contribution given by the classical lumped capacity method. By introducing the solution for $\theta_0(s)$ into Eq. (20), we obtain that

$$\frac{\partial^2\theta_1}{\partial\chi^2} - \frac{\partial\theta_1}{\partial\sigma} = \exp(-s) - 1 \tag{28}$$

and proposing a solution of the form $\theta_1(s, \sigma, \chi) = \theta_0(s) \times [h(\chi) + g(\chi, \sigma)]$, can easily to be showed that $h(\chi) = -\chi^2/2$ and the function $g(\sigma, \chi)$ satisfies the following basic problem:

$$\frac{\partial^2 g}{\partial\chi^2} - \frac{\partial g}{\partial\sigma} = 0 \tag{29}$$

with the initial and boundary conditions

$$g(\chi, 0) = -\frac{1}{2}\chi^2, \qquad \left.\frac{\partial g}{\partial\chi}\right|_{\chi=0,1} = 0 \tag{30}$$

The solution of (29) is easily obtained through Fourier transform and can be written as

$$g(\sigma, \chi) = \frac{1}{6} + \sum_{n=1}^{\infty} \frac{2}{(n\pi)^2}(-1)^n \times \cos(n\pi\chi)\exp\left[-(n\pi)^2\sigma\right] \tag{31}$$

Therefore, the solution to (20) is given as

$$\theta_1(s,\sigma,\chi) = \left[1-\exp(-s)\right] \times \left\{-\frac{1}{2}\chi^2 + \frac{1}{6} + \sum_{n=1}^{\infty}\frac{2}{(n\pi)^2}(-1)^n \times \cos(n\pi\chi)\exp\left[-(n\pi)^2\sigma\right]\right\} \tag{32}$$

Repeating again the procedure and after integration of (21) from $\chi = 0$ to $\chi = 1$, we obtain

$$-\frac{\partial}{\partial\kappa}\int_0^1 \theta_2\,\mathrm{d}\chi = \exp(-s)\left[\omega_1 + \frac{1}{3}\right] \tag{33}$$

From Eq. (33), the left-hand side has to be zero in order to avoid the appearance of secular terms, thus obtaining the value of ω_1 as $\omega_1 = -1/3$.

Finally, the nondimensional wall temperature in terms of the original nondimensional variables χ and τ and up to terms of order Bi can be written as

$$\theta(\chi,s,\sigma\colon B_i) = \left[1-\exp(-s)\right] \times \left\{1 + Bi\left(-\frac{1}{2}\chi^2 + \frac{1}{6} + \sum_{n=1}^{\infty}\frac{2}{(n\pi)^2}(-1)^n \times \cos(n\pi\chi)\exp\left[-(n\pi)^2\sigma\right]\right)\right\} \tag{34}$$

with the corresponding time scales s and σ given by Eq. (18). From Eq. (34) easily is derived the asymptotic steady-state solution, θ_{ss} valid for $\tau \to \infty$:

$$\theta_{ss} = 1 + Bi\left(-\frac{1}{2}\chi^2 + \frac{1}{6}\right) \tag{35}$$

4. Results and summary

In this paper we have analytically solved the unsteady heat conduction process due to a uniform heat generation in a solid slab, $\dot{q}$. The resulting transient temperature profile up to terms of order Bi, serves to evaluate the corresponding nondimensional entropy generation rate per unit volume, using the nondimensional relationship of the theorem of entropy generation

$$\Phi_s = \frac{\beta}{(1+\beta\theta)^2}\left(\frac{\mathrm{d}\theta}{\mathrm{d}\chi}\right)^2$$

and with the aid of this relationship, we evaluate numerically the nondimensional spatial and global averages of the entropy generation rate per unit volume Φ and Ψ, respectively.

In the limit of small values of Biot numbers was derived the nondimensional energy equation of the solid slab, emphasizing the singular behavior of this equation. In this case, we introduce two appropriate time scales s and σ, and the unsteady heat conduction process with internal heat generation is solved for the temperature profile with the aid of multiple-scale perturbation technique.

The main results are plotted following this sequence: for each fixed value of the Biot number Bi ($= 0.001$, 0.01 and 0.1), and varying the nondimensional time τ ($= 1.0, 1.1, 1.2, 1.3, \ldots$), the analytical solution of the slab temperature $\theta = \theta(\chi, \tau\colon Bi)$ are evaluated. In this form we have plotted Figs. 2–4. These figures show clearly the influence of the Biot number: for increasing values of Bi, the spatial influence is going to be more relevant and just by $Bi = 0.1$ the percentile temperature differences between middle plain of the solid slab and the external surface are of order of $\Delta\theta \sim 5\%$. Otherwise, for small values of the Biot number, the classical lumped solution is clearly revealed, because the spatial variations of the temperature are insignificant as is appreciated from Fig. 2. Therefore, the

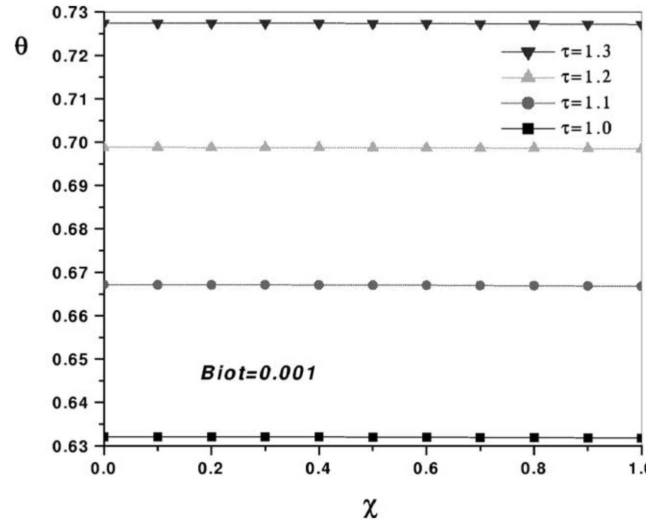

Fig. 2. Temporal evolution of the nondimensional temperature profile θ, as a function of the nondimensional coordinate χ with $Bi = 0.001$.

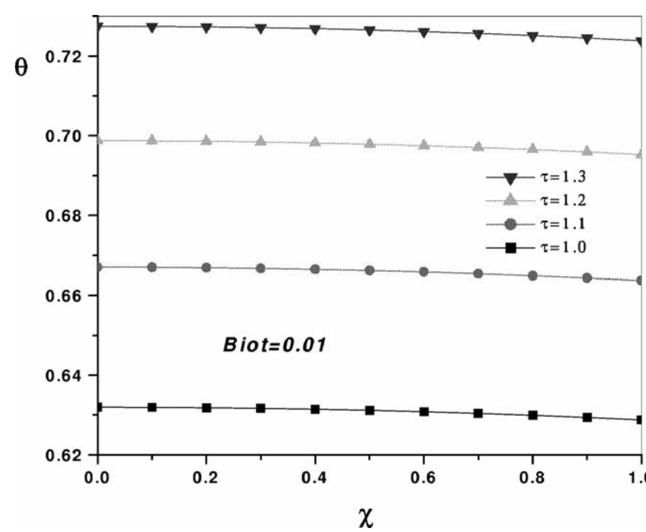

Fig. 3. Temporal evolution of the nondimensional temperature profile θ, as a function of the nondimensional coordinate χ with $Bi = 0.01$.

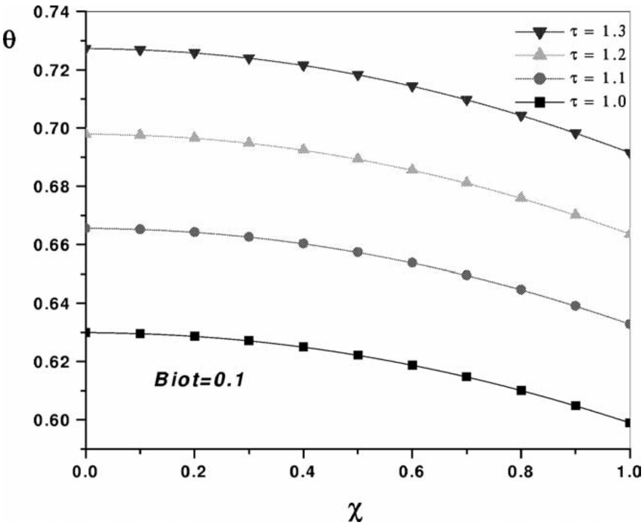

Fig. 4. Temporal evolution of the nondimensional temperature profile θ, as a function of the nondimensional coordinate χ and $Bi = 0.1$.

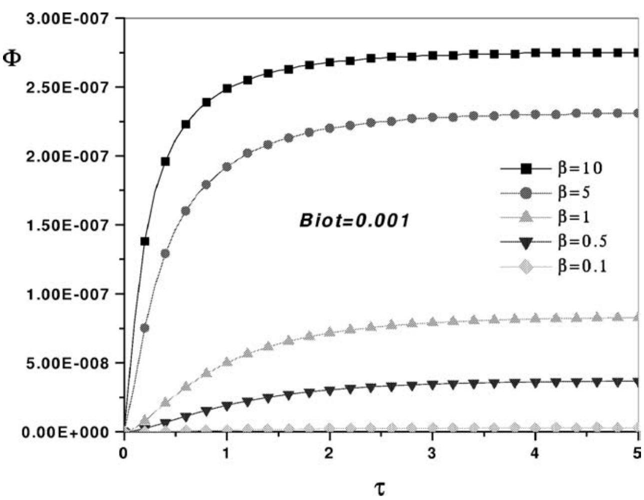

Fig. 5. Spatial average of the nondimensional entropy generation rate Φ, for different values of the nondimensional parameter β and $Bi = 0.001$.

simple solution for the nondimensional temperature given by Eq. (34) reflects very well the influence of the time scales, recognizing that the Biot number is the ratio of two time scales: $Bi = \frac{t_D}{t_C} = \frac{hL}{\lambda}$.

With the above results, we generate Figs. 5–8, where the nondimensional spatial average entropy generation rate per unit volume, $\Phi = \Phi(\tau, Bi, \beta)$, is plotted as a function of the time scale τ, different values of the internal heat generation parameter β and for each figure a selected value of the Biot number, Bi. In these figures the spatial average entropy generation rate per unit volume Φ is always an increasing function. In this sense, during the transient step the corresponding values of Φ are only bounded by the nondimensional parameters Bi and β: for increasing values of the Biot number Bi, the corresponding transient values of the function Φ are increased. However, if we increase the values of the parameter β, also Φ is increased. This means

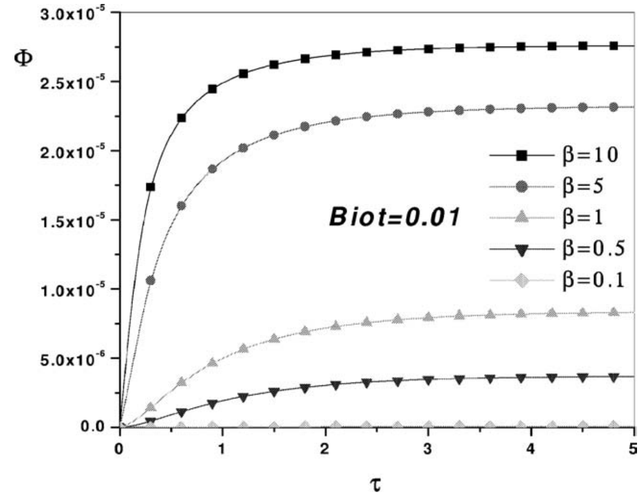

Fig. 6. Spatial average of the nondimensional entropy generation rate Φ, for different values of the nondimensional parameter β and $Bi = 0.01$.

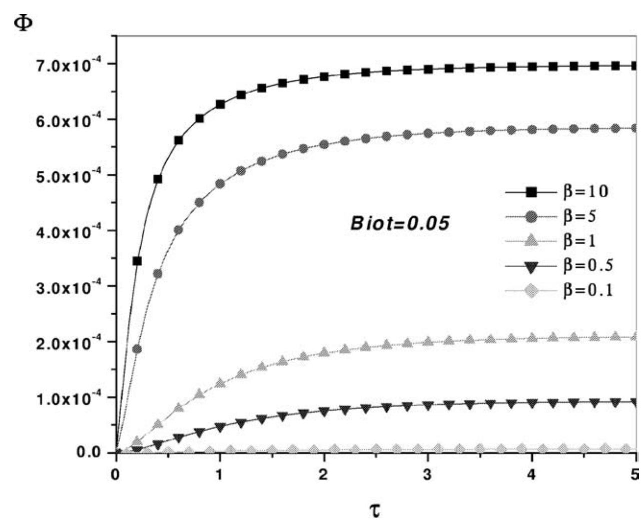

Fig. 7. Spatial average of the nondimensional entropy generation rate Φ, for different values of the nondimensional parameter β and $Bi = 0.05$.

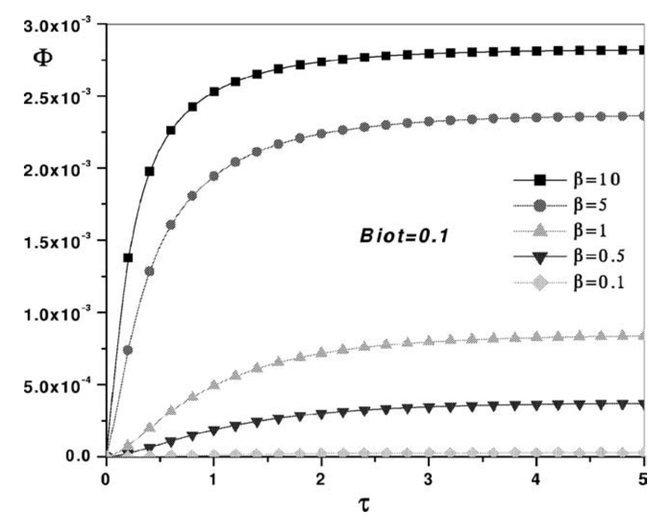

Fig. 8. Spatial average of the nondimensional entropy generation rate Φ, for different values of the nondimensional parameter β and $Bi = 0.1$.

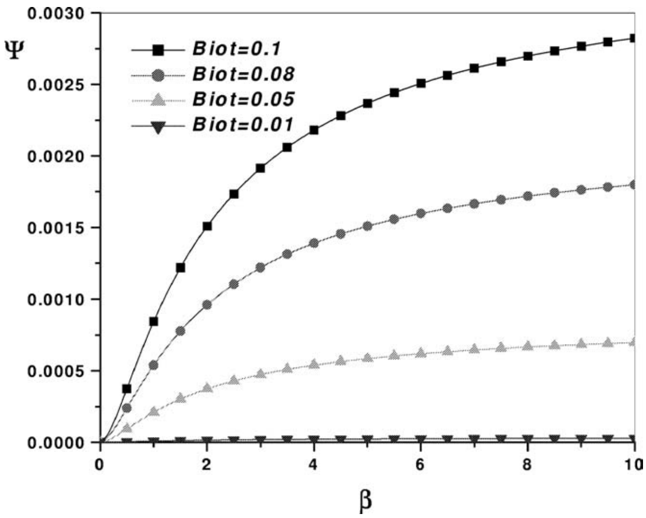

Fig. 9. Global average entropy generation rate Ψ, as a function of the nondimensional parameter β and different values of the Biot number ($Bi = 0.1$, 0.08, 0.05 and 0.01).

that both parameters play a fundamental role to show the presence of thermal irreversibilities in this transient heating process. In these figures, once the transient step is finished, the solution reaches a steady state solution denoted by Φ_{ss}. Integrating spatially this function Φ_{ss} through relationship (16), we obtain the average steady-state entropy generation rate, Ψ, which is plotted in Fig. 9 as function of the internal heat generation parameter β and different values of the Biot number, Bi. Also in this figure, we confirm the above results, indicating that both parameters increase the presence of such irreversibilities.

Finally, the developed theoretical analysis in this work provides the existence of a simple solution of the temperature profile and the associated entropy generation rates for the transient heating of a solid slab. In particular, the developed analytical procedure permits to clarify the importance of the multiple scale analysis in studies of heat conduction.

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